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Malcher, Andreas (D-GSSN-II); **Meckel, Katja** (D-GSSN-II); **Mereghetti, Carlo** (I-MILAN-IF); **Palano, Beatrice** (I-MILAN-IF)

Description complexity of pushdown store languages. (English summary)

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The pushdown store language of a (non-deterministic) pushdown automaton (PDA)

$$M = \langle Q, \Sigma, \Gamma, \delta, q_0, Z_0, F\rangle,$$

where Q is the (finite) set of states, Σ is the finite input alphabet, Γ is the finite pushdown alphabet, δ is the transition function from $Q \times (\Sigma \cup \{\lambda\}) \times \Gamma$ to finite subsets of $Q \times \Gamma^*$, $q_0 \in Q$ is the initial state, $Z_0 \in \Gamma$ is the bottom-of-pushdown symbol and $F \subseteq Q$ is the set of accepting (final) states, is the set $\mathsf{P}(M)$ of all words occurring on the pushdown store along accepting computations of M, i.e.,

$$\{u \in \Gamma^* \mid \exists x, y \in \Sigma^* \, \exists q \in Q \, \exists f \in F \, \exists \gamma \in \Gamma^* : (q_0, xy, Z_0) \vdash^* (q, y, u) \vdash^* (f, \lambda, \gamma)\}.$$

S. A. Greibach proved in 1967 that, with the above definitions, $\mathsf{P}(M)$ is a regular language [Proc. Amer. Math. Soc. **18** (1967), 263–268; MR0209086]. In the present paper, the authors take a proof of this theorem given in [J.-M. Autebert, J. Berstel and L. Boasson, in *Handbook of formal languages, Vol. 1*, 111–174, Springer, Berlin, 1997; MR1469995] and note that it gives the upper bound

$$|Q|^3|\Gamma| + |Q|^2(|\Gamma| + 1) + |Q| + 1$$

for the size (number of states) of an NFA accepting $\mathsf{P}(M)$. Then by improving the proof they obtain the upper bound

$$|Q|^2(|\Gamma| + 1) + |Q|(2|\Gamma| + 3) + 2$$

and show its optimality by proving the existence of an NFA for accepting $\mathsf{P}(M)$ with $O(|Q|^2|\Gamma|)$ states, and the existence of infinitely many PDAs denoted $M_{Q,\Gamma}$ such that any NFA accepting $\mathsf{P}(M_{Q,\Gamma})$ needs $\Omega(|Q|^2|\Gamma|)$ many states. Then some special PDAs, namely PDAs which never pop, stateless PDAs and counter-machines (PDAs with $\Gamma = \{Z_0, Z\}$) are studied, and at the end the **P**-completeness of (the decidability of) the following problems is proved for a given PDA M:

(1) if $\mathsf{P}(M)$ is finite;
(2) if $\mathsf{P}(M)$ is a finite set of words having at most length k for a given $k \geq 1$;
(3) if $\mathsf{P}(M)$ is a subset of $Z^* Z_0$.

Saeed Salehi