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On torsion in finitely presented groups. (English summary)
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A key technical observation of this paper is that the Higman embedding theorem (that every recursively presented group embeds into a finitely presented group) can preserve the set of orders of torsion elements; this is stated as the following theorem, in which

$$\mathrm{Tor}(G) = \{g \in G \mid g \text{ is torsion}\} \quad \text{and}$$
$$\mathrm{Tord}(G) = \{n \in \mathbb{N} \mid \exists g \in \mathrm{Tor}(G) \text{ with } o(g) = n \geq 2\}.$$

Theorem 2.2. There is a uniform algorithm that, on input of a countably generated recursive presentation $P = \langle X \mid R\rangle$, constructs a finite presentation $\mathrm{T}(P)$ such that $\overline{P} \hookrightarrow \overline{\mathrm{T}(P)}$ and $\mathrm{Tord}(\overline{P}) = \mathrm{Tord}(\overline{\mathrm{T}(P)})$, along with an explicit embedding $\overline{\phi}\colon \overline{P} \hookrightarrow \overline{\mathrm{T}(P)}$.

The author notes that every group has a unique torsion-free quotient through which all other torsion-free quotients factor:

Corollary 3.4. If G is a group, then $G/\mathrm{Tor}_\infty(G) = G^{\mathrm{tf}}$, which is the torsion-free universal quotient for G.

Here, $\mathrm{Tor}_0(G) = \{e\}$, $\mathrm{Tor}_{n+1} = \langle\langle\{g \in G \mid g\,\mathrm{Tor}_n(G) \in \mathrm{Tor}(G/\mathrm{Tor}_n(G))\}\rangle\rangle^G$, and $\mathrm{Tor}_\infty(G) = \bigcup_{i\in\mathbb{N}} \mathrm{Tor}_i(G)$. "By standard techniques in combinatorial group theory, we show … the existence of an algorithm that takes any finite presentation P and outputs a recursive presentation P^{tf} of the torsion-free universal quotient of $\overline{P}$":

Proposition 3.8. There is a uniform algorithm that, on input of a countably generated recursive presentation $P = \langle X \mid R\rangle$ of a group $\overline{P}$, outputs a countably generated recursive presentation $P^{\mathrm{tf}} = \langle X \mid R'\rangle$ (on the same generating set X, and with $R \subseteq R'$ as sets) such that $\overline{P^{\mathrm{tf}}}$ is the torsion-free universal quotient of $\overline{P}$, with associated surjection given by extending $\mathrm{id}_X\colon X \to X$.

Then the main result of the paper "follows by combining Theorem 2.2 and Proposition 3.8, in a similar way to Higman's original construction of a universal finitely presented group":

Theorem 3.10. There is a universal finitely presented torsion-free group G. That is, G is torsion-free, and for any finitely presented group H, we have that $H \hookrightarrow G$ if (and only if) H is torsion-free.

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