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Tyszka, Apoloniusz (PL-AGKRPR)
A new characterization of computable functions. (English summary)
An. Ştiinţ. Univ. "Ovidius" Constanţa Ser. Mat. **21** (2013), *no. 3,* 289–293.

The title of this paper could be misleading, since there is no (new) "characterization" of computable functions in it; rather, a couple of new properties of computable functions are proved:

Theorem 1. There is an algorithm which accepts as input any computable function $f\colon \mathbb{N}\to\mathbb{N}$ and returns a positive integer $m(f)$ and a computable function g which to any integer $n\geqslant m(f)$ assigns a system

$$S\subseteq\{x_i=1\mid 1\leqslant i\leqslant n\}\cup\{x_i+x_j=x_k\mid 1\leqslant i,j,k\leqslant n\}\cup\{x_i\cdot x_j=x_k\mid 1\leqslant i,j,k\leqslant n\}$$

such that S is satisfiable over integers and each integer tuple $(x_1,\dots,x_n)$ that solves S satisfies $x_1=f(n)$.

Theorem 2. There is an algorithm which accepts as input any computable function $f\colon \mathbb{N}\to\mathbb{N}$ and returns a positive integer $w(f)$ and a computable function h which to any integer $n\geqslant w(f)$ assigns a system

$$S\subseteq\{x_i=1\mid 1\leqslant i\leqslant n\}\cup\{x_i+x_j=x_k\mid 1\leqslant i,j,k\leqslant n\}\cup\{x_i\cdot x_j=x_k\mid 1\leqslant i,j,k\leqslant n\}$$

such that S is satisfiable over non-negative integers and each tuple $(x_1,\dots,x_n)$ of non-negative integers that solves S satisfies $x_1=f(n)$.

Reading two earlier papers of the author [Inform. Process. Lett. **113** (2013), no. 19-21, 719–722; MR3095449; Fund. Inform. **125** (2013), no. 1, 95–99; MR3114060] can be very helpful for understanding the new paper and seeing the results in perspective.

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