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Kikuchi, Makoto [[Kikuchi, Makoto¹](#)] (J-KOBEE-GSI);

Kurahashi, Taishi (J-KOBEE-GSI);

Sakai, Hiroshi [[Sakai, Hiroshi¹](#)] (J-KOBEE-GSI)

On proofs of the incompleteness theorems based on Berry's paradox by Vopěnka, Chaitin, and Boolos. (English summary)

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A length diagonalization lemma is proved in the paper, and based on it some proofs for the first and second incompleteness theorems (of K. Gödel) are presented. This shows that a proof based on Berry's paradox, as are the proofs of Boolos, Chaitin and Vopěnka (which do not use a diagonal argument), can be carried out using the diagonal method. Also a notion of Kolmogorov complexity is introduced in the paper which is based on provability in an arithmetical theory rather than computability by some Turing machine. This enables the authors to obtain a Chaitin-like proof for the incompleteness phenomenon. Finally, Vopěnka's proof of the second incompleteness theorem, which was based on set theoretic notions, is imported to arithmetical theories by using the arithmetized completeness theorem.

The paper is a good read and contains very interesting results. Unfortunately there are several misprints and mistakes; also the English of the paper needs major revision.

Saeed Salehi

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.