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Fixed points of endomorphisms over special confluent rewriting systems. (English summary)

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From the text: "Cayley graphs of monoids defined through special confluent rewriting systems are known to be hyperbolic metric spaces which admit a compact completion given by irreducible finite and infinite words. In this paper, we prove that the fixed point submonoids for endomorphisms of these monoids which are boundary injective (or have bounded length decrease) are rational, with similar results holding for infinite fixed points. Decidability of these properties is proved, and constructibility is proved for the case of bounded length decrease. These results are applied to free products of cyclic groups, providing a new generalization for the case of infinite fixed points.

"The present paper intends to prove finite generation properties for both finite and infinite fixed points. This is achieved through a combination of automata-theoretic, combinatorial and topological techniques. Two classes of endomorphisms are studied: boundary-injective endomorphisms and endomorphisms with bounded length decrease. The first class provides new proofs for the already known results for monomorphisms of free groups and more generally free products of cyclic groups. The second class provides constructibility results that are reminiscent of those of O. S. Maslakova [Algebra Logika **42** (2003), no. 4, 422–472, 510–511; MR2017513 (2004i:20044)]. Moreover, both classes are recursive and algorithms to test the corresponding properties are provided.

"We hope this paper provides some further evidence for the potential of automata-theoretic techniques in the study of dynamical problems, for monoids and for groups as well. We list now some open problems that arise naturally from this work:

(1) We have no examples of uniformly continuous endomorphisms which are not finite-splitting, nor do we know whether or not this property is decidable.
(2) It would be very desirable to have a proof that Fix φ is effectively constructible when φ is boundary-injective, providing in particular an alternative (combinatorial?) proof for Maslakova's Theorem [op. cit.], but it has eluded us so far."

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.