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Michel, Pascal (F-PARIS7-EL)

Homology of groups and third busy beaver function. (English summary)

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The paper under review gives some results on the function b_2 and computes $b_k(N)$ for some small values of N . The function b_k is defined as follows: $b_k(N)$ is the rank of the k th homology group $H_k(G)$ of maximum finite rank, among the finitely presented groups G with presentation of length no bigger than N . The length of a finite (group) presentation $\langle x_1, \dots, x_n \mid r_1, \dots, r_m \rangle$, where the x_i 's are generators and the r_j 's are relators, is $n + \sum_{1 \leq i \leq m} \text{length}(r_i)$. The function b_2 is shown to be non-computable, but computable with oracle \emptyset' . Also the functions b_q for $q \geq 3$ grow as the third busy beaver function, so they are not computable (and grow faster than any function computable) with oracle \emptyset'' . These functions were discovered as naturally defined functions that grow faster than any \emptyset' -computable function by A. Nabutovsky and S. Weinberger [Topology **46** (2007), no. 2, 211–223; MR2313072 (2008f:20065)], which solved a longstanding open problem.

Saeed Salehi

References

1. S. Aaronson, Who can name the bigger number? (1999) <http://www.scottaaronson.com/writings/bignumber>
2. W. Ackermann, Zum Hilbertschen Aufbau der reellen Zahlen, *Math. Ann.* **99** (1928) 118–133. MR1512441
3. G. Baumslag, E. Dyer and C. F. Miller III, On the integral homology of finitely presented groups, *Topology* **22** (1983) 27–46. MR0682058 (85g:20044)
4. R. Bieri, *Homological Dimension of Discrete Groups* (Queen Mary College, Mathematics Notes, 1976). MR0466344 (57 #6224)
5. G. R. Blakley and I. Borosh, Knuth's iterated powers, *Adv. Math.* **34** (1979) 109–136. MR0549780 (81e:05026)
6. W. A. Bogley and J. Harlander, Homological decision problems for finitely generated groups with solvable word problem, *Int. J. Alg. Comput.* **12** (2002) 213–221. MR1902366 (2003b:20048)
7. A. H. Brady, The determination of the value of Rado's noncomputable function $\Sigma(k)$ for four-state Turing machines, *Math. Comput.* **40** (1983) 647–665. MR0689479 (84d:03049)
8. J. H. Conway and R. K. Guy, *The Book of Numbers* (Springer, 1996). MR1411676 (98g:00004)
9. GAP system, <http://www.gap-system.org>.
10. C. McA. Gordon, Some embedding theorems and undecidability questions for groups, in *Combinatorial and Geometric Group Theory*, eds. A. J. Duncan *et al.*, London Mathematical Society Lecture Notes Series, Vol. 204 (Cambridge University Press, 1995), pp. 105–110. MR1320278 (96b:20037)
11. HAP package, <http://hamilton.nuigalway.ie/Hap/www>.
12. P. J. Hilton and U. Stammbach, *A Course in Homological Algebra* (Springer, 1971). MR0346025 (49 #10751)
13. G. Higman, Subgroups of finitely presented groups, *Proc. Roy. Soc. London Series A* **262** (1961) 455–475. MR0130286 (24 #A152)

14. D. L. Johnson, *Presentations of Groups* (Cambridge University Press, 1990). [MR1056695 \(91h:20001\)](#)
15. S. C. Kleene, *Introduction to Metamathematics* (Van Nostrand, 1952). [MR0051790 \(14,525m\)](#)
16. D. E. Knuth, Mathematics and computer science: Coping with finiteness, *Science* **194** (1976) 1235–1242. [MR0534161 \(84i:68002a\)](#)
17. Yu. Kuz'min, Homological methods in group theory, *CRM Proc. Lect. Notes* **17** (1999) 81–98. [MR1653686 \(2000a:20115\)](#)
18. S. Lin and T. Rado, Computer studies of Turing machine problems, *J. ACM* **12** (1965) 196–212. [MR0195649 \(33 #3847\)](#)
19. R. C. Lyndon and P. E. Schupp, *Combinatorial Group Theory* (Springer, 1977). [MR0577064 \(58 #28182\)](#)
20. H. Marxen, <http://www.dr.b.insel.de/~heiner/BB>.
21. P. Michel, Busy beaver competition and Collatz-like problems, *Arch. Math. Logic* **32** (1993) 351–367. [MR1223396 \(94f:03048\)](#)
22. P. Michel, Small Turing machines and generalized busy beaver competition, *Theoret. Comput. Sci.* **326** (2004) 45–56. [MR2094241 \(2005e:68049\)](#)
23. P. Michel, <http://www.logique.jussieu.fr/~michel>, P. Michel, The busy beaver competition: A historical survey, <http://arxiv.org/abs/0906.3749>. [MR0792488 \(86f:03003\)](#)
24. C. F. Miller III, Decision problems for groups—survey and reflections, in *Algorithms and Classification in Combinatorial Group Theory*, eds. G. Baumslag and C. F. Miller III, MSRI, Vol. 23 (1992), pp. 1–59. [MR1230627 \(94i:20057\)](#)
25. A. Nabutovsky and S. Weinberger, Betti numbers of finitely presented groups and very rapidly growing functions, *Topology* **42** (2007) 211–223. [MR2313072 \(2008f:20065\)](#)
26. T. Rado, On non-computable functions, *Bell System Tech. J.* **41** (1962) 877–884. [MR0133229 \(24 #A3063\)](#)
27. J. J. Rotman, *An Introduction to Homological Algebra*, 2nd edn. (Springer, 2009). [MR2455920 \(2009i:18011\)](#)
28. R. I. Soare, *Recursively Enumerable Sets and Degrees* (Springer, 1987). [MR0882921 \(88m:03003\)](#)
29. E. H. Spanier, *Algebraic Topology* (McGraw-Hill, 1966). [MR0210112 \(35 #1007\)](#)
30. J. Stallings, A finitely presented group whose 3-dimensional integral homology is not finitely generated, *Amer. J. Math.* **85** (1963) 541–543. [MR0158917 \(28 #2139\)](#)
31. C. A. Weibel, *An Introduction to Homological Algebra* (Cambridge University Press, 1994). [MR1269324 \(95f:18001\)](#)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.