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Kripke semantics for provability logic GLP. (English summary)

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The paper solves some open questions about the polymodal provability logic **GLP** introduced by G. Japaridze in 1986. The logic describes all the universally valid schemata for the reflection principles of restricted logical complexity in arithmetic, in the style of provability logic. A complete Kripke-style semantics for **GLP** is introduced in the paper; this allows some modal-logical properties of **GLP**, such as decidability and the Craig interpolation property, to be established by finitary methods. For doing so a subsystem **J** of **GLP** is singled out; this system, being a good approximation to **GLP**, is complete with respect to a natural class of finite Kripke frames. Note that **GLP** is not complete with respect to any class of Kripke frames. Two classes of Kripke models are considered for this subsystem: **J**-frames and so-called stratified frames. Then by a blow-up operation that can be applied to any finite stratified frame, one can get models for arbitrarily large fragments of **GLP**. This yields a weak completeness result for **GLP**; for obtaining a stronger completeness result such models are glued together. The author finishes the paper with two open questions about the optimal complexity of the decision procedure for **GLP**, and whether **GLP** belongs to PSpace; then he notes in a footnote that the latter question has recently been answered positively by I. Shapirovsky (2008). *Saeed Salehi*

References

1. L. Beklemishev, J. Joosten, M. Vervoort, A finitary treatment of the closed fragment of Japaridze's provability logic, *Journal of Logic and Computation* 15 (4) (2005) 447–463. [MR2157727 \(2006i:03094\)](#)
2. L.D. Beklemishev, Provability algebras and proof-theoretic ordinals, I, *Annals of Pure and Applied Logic* 128 (2004) 103–123. [MR2060550 \(2005e:03129\)](#)
3. L.D. Beklemishev, Reflection principles and provability algebras in formal arithmetic, *Uspekhi Matematicheskikh Nauk* 60 (2) (2005) 3–78 (in Russian) English translation in: *Russian Mathematical Surveys*, 60(2): 197–268, 2005. [MR2152943 \(2006e:03090\)](#)
4. L.D. Beklemishev, On the Craig interpolation and the fixed point property for GLP. *Logic Group Preprint Series* 262, University of Utrecht, December 2007. <http://preprints.phil.uu.nl/lgps/>.
5. P. Blackburn, M. de Rijke, Y. Venema, *Modal Logic*, Cambridge University Press, Cambridge, 2001. [MR1837791 \(2003b:03001\)](#)
6. G. Boolos, *The Logic of Provability*, Cambridge University Press, Cambridge, 1993. [MR1260008 \(95c:03038\)](#)
7. A. Chagrov, M. Zakharyashev, *Modal Logic*, Clarendon Press, Oxford, 1997. [MR1464942 \(98e:03021\)](#)
8. R.I. Goldblatt, *Metamathematics of modal logic*, Part I, *Reports on Mathematical Logic* 6 (1976) 41–78. [MR0536321 \(58 #27331a\)](#)
9. R.I. Goldblatt, *Metamathematics of modal logic*, Part II, *Reports on Mathematical Logic* 7 (1976) 21–52. [MR0536322 \(58 #27331b\)](#)
10. T. Icard, III, *Models of the polymodal provability logic*. M.Sc. Thesis, ILLC, Univer-

- sity of Amsterdam, <http://www.illc.uva.nl/Publications/ResearchReports/MoL-2008-06.text.pdf>, 2008.
11. K.N. Ignatiev, The closed fragment of Dzhaparidze's polymodal logic and the logic of Σ_1 conservativity, in: ITLI Prepublication Series X-92-02, University of Amsterdam, 1992.
 12. K.N. Ignatiev, On strong provability predicates and the associated modal logics, *The Journal of Symbolic Logic* 58 (1993) 249–290. [MR1217189 \(94f:03021\)](#)
 13. G.K. Japaridze, The modal logical means of investigation of provability. Thesis in Philosophy, Moscow, 1986 (in Russian).
 14. G.K. Japaridze, The polymodal logic of provability, in: *Intensional Logics and Logical Structure of Theories: Material from the fourth Soviet-Finnish Symposium on Logic*, Telavi, May 20–24, 1985, Metsniereba, Tbilisi, 1988. pp. 16–48 (in Russian). [MR1063489 \(91k:03038\)](#)
 15. I. Shapirovsky, PSPACE-decidability of Japaridze's polymodal logic, in: C. Areces, R. Goldblatt (Eds.), *Advances in Modal Logic*, vol. 7, King's College Publications, 2008, pp. 289–304. [MR2642650](#)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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