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The existence of a near-unanimity term in a finite algebra is decidable. (English summary)

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An operation f is called near-unanimity if it satisfies the following identities:

$$f(y,x,\dots,x)=f(x,y,x,\dots,x)=\cdots=f(x,\dots,x,y)=x.$$

If a finite algebra of finite signature has a near-unanimity term operation, then it has a finite base of equations. B. A. Davey, L. Heindorf, and R. McKenzie asked in 1995 whether it is decidable if a finite algebra has a near-unanimity term operation. It was already known that if a finite algebra has a near-unanimity term operation then it admits a natural duality. The converse was proved for the case of algebras in a congruence distributive variety. It is decidable whether a finite algebra lies in a congruence distributive variety. McKenzie proved in 1997 (unpublished) that it is undecidable for a given finite algebra and its two fixed elements whether there is a term operation that satisfies the above identities for those two elements. This was extended by the author in 2007 by showing that for a given finite algebra and its two fixed elements it is undecidable whether there is a term operation that satisfies the above identities for all but those two elements of the algebra.

In the present paper, it is shown that having a near-unanimity term operation is decidable for finite algebras. This is a surprising theorem after the above negative partial results. For an n-element set there are finitely many algebras with basic operations at most r-ary, and it is proved that there exists a recursive (upper) bound $N(n,r)$ for the arity of near-unanimity term operations (if any). The author mentions that currently there is no formula for that function. *Saeed Salehi*

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