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The hydra battle revisited. (English summary)

Rewriting computation and proof, 1–27, *Lecture Notes in Comput. Sci.*, 4600, Springer, Berlin, 2007.

A version of the Hydra and Hercules battle is studied in the paper. Mathematically speaking, a hydra is an unordered, rooted, finite tree and Hercules fights it by cutting a leaf (head) at a time. Hydra then regenerates according to some rules; different rules correspond to different variants of the battle. Hercules finally prevails in all the variants by cutting all heads of Hydra (making the tree empty). The variant studied in the paper follows the following leaf-generating rule: if the severed leaf had a grandparent node, then the branch issuing from that node together with the mutilated subtree is multiplied by a certain factor. The certain multiplication factor is one more than the stage of the battle in the standard versions, but the authors set it equal to the stage of the battle. This modification, as the authors claim, does not affect the results of L. Kirby and J. Paris [*Bull. London Math. Soc.* **14** (1982), no. 4, 285–293; [MR0663480 \(83j:03096\)](#)], one of which is that every strategy of Hercules is a winning strategy. This true proposition cannot be proven in PA, Peano's Arithmetic. However, if the multiplication factor is bounded, then PA can prove the winning proposition.

The authors present the battle in a functional language, as a `while` program, and as a rewriting system. A great effort (amounting to about half of the paper) has been put on proving the termination of the rewriting system, noting that to the best knowledge of the authors no full and correct termination proof has been provided in the literature. In addition to proving the termination of the rewriting system for the authors' variant of the Hydra battle, they also show that the system is not simply terminating, meaning that its termination cannot be proven by a reduction order that enjoys the subterm property (orders like the recursive path order in which subterms are smaller than the terms). Thus, one sees the difficulty of the termination, and an explanation for the lack of a complete termination proof elsewhere. This surely gives the paper a high value.

{For the entire collection see [MR2367389 \(2008j:68005\)](#)}

Reviewed by *Saeed Salehi*