

**MR2278078 (2007i:68066)** [68Q45](#) (06A06)

**Petković, Tatjana**

**Regular tree languages and quasi orders. (English summary)**

*Acta Cybernet.* **17** (2006), *no. 4*, 811–823.

The paper under review is a continuation of the author's joint "fundamental study" paper with the reviewer [*Theoret. Comput. Sci.* **347** (2005), no. 1-2, 1–35; [MR2184827 \(2006k:68115\)](#)] in which, among other results, a correspondence between families of tree languages and varieties of ordered algebras was established. The corresponding families of tree languages were called positive varieties as they satisfy all the closure properties of the standard varieties except of being closed under the complementations.

In this paper, the author adds a third dimension to the above-mentioned correspondence. Namely, positive varieties of tree languages and varieties of ordered algebras are characterized by classes of compatible well quasi orders. A quasi order is any reflexive and transitive relation; it is called a well quasi order if each infinite sequence contains an infinite ascending subsequence; and it is called compatible when it preserves the operations of the term algebra under consideration. A family of well quasi orders is a filter (i.e., closed under intersection and supersets) which is closed under the inverse images of homomorphisms.

This is analogous to D. Thérien's extension of S. Eilenberg's variety theorem (for word languages and monoids/semigroups). Note also that the variety theorem of M. Steinby and J. Almeida for tree languages and algebras also includes a correspondence with the varieties of congruences. Indeed the intersection of a compatible well quasi order with its inverse is a congruence relation, and the original quasi order induces a (partial) order on the congruence classes.

Reviewed by *Saeed Salehi*