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An algebraic characterization of temporal logics on finite trees. I. (English summary)

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This is the first part of a series of papers [see also Part II, Z. Ésik, in *Proceedings of the 1st International Conference on Algebraic Informatics*, 79–99, Aristotle Univ. Thessaloniki, Thessaloniki, 2005; MR2186456 (2006j:03019); Part III, in *Proceedings of the 1st International Conference on Algebraic Informatics*, 101–110, Aristotle Univ. Thessaloniki, Thessaloniki, 2005; MR2186457 (2006j:03017)] on CTL-like temporal logics on finite trees. A journal version of this conference paper has also appeared [Theoret. Comput. Sci. **356** (2006), no. 1-2, 136–152].

Certain CTL modalities like 'next' and 'ef' are defined in a uniform way by assigning a modal operator to each regular tree language of a given (literal) variety. Using these operations some extended (propositional) temporal logics that can have finite trees as their models are constructed.

Following the paper's notation, for a ranked alphabet Σ, the set of Σ-formulas consists of unary predicates p_σ for each $\sigma \in \Sigma$, interpreted to be true in a tree when its root is labelled with σ, and is closed under the Boolean connectives and the modal operator $L(\delta \mapsto \varphi_\delta)_{\delta\in\Delta}$, where Δ is a ranked alphabet of the same type as Σ, L is a regular Δ-tree language, and $\{\varphi_\delta\}_{\delta\in\Delta}$ is a family of Σ-formulas. For a given variety $\mathcal{L}$ of tree languages, let **FTL**($\mathcal{L}$) be the class of all tree languages definable by the aforementioned formulas, where L is taken from $\mathcal{L}$.

For any variety $\mathcal{L}$, **FTL**($\mathcal{L}$) is a variety that contains $\mathcal{L}$; indeed **FTL** is a closure operation. It is shown that **FTL** of the next modality is the variety $\mathcal{D}$ of definite tree languages, and more generally, the next modality is expressible by **FTL**($\mathcal{L}$) iff **FTL**($\mathcal{L}$) contains $\mathcal{D}$. By a variety theorem, **FTL**($\mathcal{L}$) corresponds to a variety of finite algebras (automata) which can be shown to be closed under the cascade product when next is expressible in **FTL**($\mathcal{L}$).

The main results of the paper provide algebraic characterizations for **FTL**-definable varieties; for example, the variety of tree languages definable by next and ef modalities corresponds to the variety of finite algebras that contains $\mathbb{E}_F$ and $\mathbb{D}_0$ and is closed under the cascade product; here $\mathbb{E}_F$ and $\mathbb{D}_0$ are certain two-element algebras. Thus, a main contribution of the paper is to reduce the problem of CTL-definability of a tree language to the membership problem of a variety of finite algebras. To make this reduction effective, one has to develop a structure theory of finite algebras. This is done in Part III for next+ef.

There are several misprints and mistakes in the paper, some of which have been corrected in the above-cited journal version of the paper.

{For the entire collection see MR2184982 (2006f:68005)}

Reviewed by *Saeed Salehi*