MR2020045 (2004j:03078) 03F30 (03F55 03H15)
Moniri, Morteza (IR-TPM)
Intuitionistic weak arithmetic. (English summary)
Arch. Math. Logic **42** (2003), *no. 8,* 791–796.

For a class of arithmetical formulas Γ, $i\Gamma$ is the intuitionistic version of the induction schema $I\Gamma$. Similarly, $l\Gamma$ is the intuitionistic arithmetic axiomatized by the least number principles for the formulas in Γ. The axiom AEO states that every element is either even or odd, that is, the totality of $\lceil x/2 \rceil$, and $\exp$ states the totality of the exponential function 2^x.

Some weak fragments of Heyting Arithmetic HA are studied in the paper by Kripke model-theoretic methods. The already known results $i\forall_1 \nvdash AEO$ and $i\Pi_1 \nvdash \exp$ are enhanced by constructing two ω-framed Kripke models for $i\forall_1 + \neg AEO$ and $i\Pi_1 + \neg\exp$. It is shown that $i\forall_1$ (respectively $i\Pi_1$) is not closed under the Double Negation Rule for $\exists_1$-formulas $DNS(\exists_1)$ (respectively for Σ_1-formulas $DNS(\Sigma_1)$), and the double negation of the least number principle for open formulas cannot be proved within $i\forall_1$, that is, $i\forall_1 \nvdash \neg\neg l\mathrm{op}$; moreover $i\Pi_1 \nvdash \neg\neg i\Sigma_1$. The well-known fact that HA does not imply the Least Number Principle is sharpened by showing $HA \nvdash l\Sigma_1$.

There is a little inessential error on page 794: Wehmeier's result $i\Pi_1 \nvdash \exp$ [K. F. Wehmeier, Arch. Math. Logic **37** (1997), no. 1, 37–49; MR1485862 (99a:03062)] is not proved by constructing a two-node Kripke model; his proof is "by contradiction" and rather non-constructive. Also the proof of Proposition 1.5 is not correct. Nevertheless, it can be proved by the following argument: Let $T \supseteq i\Delta_0 + \exp$, and let $M \models T^c$ be a non-standard model. Take a non-standard $a \in M$ and put $I = a^{\mathbb{N}}$. Then the two-node Kripke model constructed by putting M on top of I forces $\neg\neg T^i$ but does not force T^i, since its root does not satisfy $\exp$. Hence $\neg\neg T^i \nvdash T^i$. Apart from that, the rest of the paper is well-written and worth reading.

Reviewed by *Saeed Salehi*

References

1. Ardeshir, M., Moniri, M.: Intuitionistic open induction and open least number principle and the buss operator. Notre Dame J. Formal Logic **39**, 212–220 (1998) MR1714956 (2000h:03116)
2. Burr, W.: Fragments of heyting arithmetic. J. Symbolic Logic **63**, 1223–1240 (2000) MR1791374 (2001m:03120)
3. Buss, S.: Intuitionistic validity in T-normal kripke structures. Ann. Pure Appl. Logic **59**, 159–173 (1993) MR1213270 (94m:03020)
4. Dragalin, A.G.: Mathematical intuitionism. Introduction to proof theory. Translated from the Russian by Elliott Mendelson. Translation edited by Ben Silver. Translations of Mathematical Monographs, 67. American Mathematical Society, Providence, RI, 1988 MR0932457 (90f:03001)
5. Hajek, P., Pudlak, P.: Metamathematics of First-order Arithmetic. Springer-Verlag, Berlin, 1993

MR1219738 (94d:03001)
6. Kaye, R.: Models of Peano Arithmetic. Oxford University Press, Oxford, 1991 MR1098499 (92k:03034)
7. Moniri, M., Moniri, M.: Some weak fragments of HA and certain closure properties. J. Symbolic Logic **67**, 91–103 (2002) MR1889538 (2002k:03102)
8. Moniri, M.: Weak arithmetics and kripke models. Math. Logic Quart. **48**, 157–160 (2002) MR1874213 (2002m:03092)
9. Moniri, M.: Independence results for weak systems of intuitionistic arithmetic. Math. Logic Quart. **49**, 250–254 (2003) MR1979130 (2004d:03129)
10. Smorynski, C.: Applications of Kripke Models. In: Metamathematical Investigations of Intuitionistic Arithmetic and Analysis (A.S. Troelstra, ed.), Springer Lecture Notes in Mathematics, Vol. 344, Springer-Verlag, Berlin, 1973 MR0444442 (56 #2795)
11. Troelstra, A.S., van Dalen, D.: Constructivism in Mathematics. Vol. I, North-Holland, Amsterdam, 1988 MR0970277 (90e:03002b)
12. Wehmeier, K.F.: Fragments of HA based on Σ_1-induction. Arch. Math. Logic **37**, 37–49 (1997) MR1485862 (99a:03062)
13. Wehmeier, K.F.: Constructing kripke models of certain fragments of heyting's arithmetic. Publ. Inst. Math. (Beograd) (N.S.) **77**, 1–8 (1998) MR1633666 (2000d:03154)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.