Variants of Kleene's Recursive Realizability

Saeed Salehi

Institute of Mathematics, Polish Academy of Science

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The notion of Recursive Realizability introduced by Kleene is a useful tool for measuring the strength of intuitionistic arithmetics. Various generalizations of this realizability has been proposed by restricting the class of recursive functions to a proper sub-class. As such examples one could mention Damnjanovic's or López-Escobar's primitive recursive realizability or Plisko's Σ_n–realizability.

In this talk, I present definitions and basic properties of two realizabilities introduced by the speaker, namely that of realizability by primitive recursive functions and realizability by polynomially bounded (primitive) recursive functions. Applying these to Ruitenburg's Basic Arithmetic (a sub-system of Heyting Arithmetic) yields interesting results about its provably total functions. Applicability of these realizabilities to Basic Arithmetic results from the observations that

(1) the Gödel codes of primitive recursive and polynomially bounded recursive functions are arithmetically definable, and

(2) the S-m-n functions in Recursion Theory can be chosen from the class of primitive recursive or polynomially bounded recursive functions.