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**Zbl 1168.03044****Pudlák, Pavel****Fragments of bounded arithmetic and the lengths of proofs.** (English)

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In his earlier paper [“Consistency and games – in search of new combinatorial principles”, Lect. Notes Log. 24, 244–281 (2006; Zbl 1105.03059)], the author introduced combinatorial principles  $\mathbf{A}_n$  (for  $n = 0, 1, 2, \dots$ ) and proved that  $\mathbf{A}_{n+1}$  is provable in  $T_2^n$  and every  $\forall\Sigma_1^b$  sentence of  $T_2^n$  is provable from  $S_2^1 + \mathbf{A}_{n+5}$ . In this paper, the author improves that result by showing that already  $S_2^1 + \mathbf{A}_{n+1}$  proves all  $\forall\Sigma_1^b$  sentences of  $T_2^n$ . Also some new principles, denoted by  $\mathbf{A}_n^\infty$ , are introduced, which form an increasing hierarchy in the sense that for every  $n$  there exists  $k > n$  such that  $\mathbf{A}_k^\infty$  needs exponentially more queries than  $\mathbf{A}_n^\infty$ . The author views this as an evidence supporting the still open conjecture that  $T_2^{n+1}[R]$  is not  $\forall\Sigma_1^b[R]$ -conservative over  $T_2^n[R]$ ; where  $R$  is a predicate symbol interpreted as oracle.

*Saeed Salehi (Tabriz)**Classification :*

\*03F20 Complexity of proofs

03F30 First-order arithmetic and fragments