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On the additive theory of prime numbers. (English)

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In connection to the open problem of “the decidability or undecidability of $\text{Th}(\mathbb{N}, +, \mathbb{P})$ where \mathbb{P} is the set of primes” the paper considers the problem of the decidability of $\text{Th}(\mathbb{N}, +, n \mapsto p_n)$ where p_n denotes the $(n + 1)$ -th prime number. The latter problem and the problem of whether p_n 's are definable in $(\mathbb{N}, +, \mathbb{P})$ are still open.

Since, by the prime numbers theorem, p_n can be approximated by $n \log(n)$, the author studies $\text{Th}(\mathbb{N}, +, n \mapsto n \log(n))$ and $\text{Th}(\mathbb{N}, +, n \mapsto n \log_2(n))$ and proves their undecidability. Here $\log(n)$ and $\log_2(n)$ are the integer part (floor) of Neperian logarithm and binary logarithm of n . In the second part of the paper, it is proved that multiplication is definable in $(\mathbb{N}, +, n \mapsto p_n, n \mapsto r_n)$, where r_n is the remainder of p_n divided by n , by an existential formula. Thus $\text{Th}_{\exists}(\mathbb{N}, +, n \mapsto p_n, n \mapsto r_n)$ is undecidable.

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Classification :

*11U05 Decidability related to number theory

03B25 Decidability of theories and sets of sentences