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On modal logic of deductive closure. (English)

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Let T be a fixed logical theory. For any set of formulas Σ in the language of T, the deductive closure of a subset $A \subseteq \Sigma$ with respect to (provability in) T in (the universe) Σ is defined to be the set $\{\alpha \in \Sigma \mid A \vdash_T \alpha\}$. The paper under review studies the modal logic of this deductive closure, i.e., when propositional variables are interpreted as arbitrary subsets of Σ (and $\bot$ is interpreted as $\emptyset$), Boolean connectives as usual Boolean operations on sets, and the BOX operator $\mathbf{B}$ as the above deductive closure (when ϕ is interpreted as A, then $\mathbf{B}\phi$ is interpreted as $\{\alpha \in \Sigma \mid A \vdash_T \alpha\}$). The (modal) logic of deductive closure of a theory T is, by definition, the set of (modal) formulas and rules which are valid for every set of formulas Σ and every interpretation. When T is just the classical propositional logic, the logic of deductive closure of T (called $\mathcal{D}$ in the paper) is axiomatized by the reflexivity ($\phi \to \mathbf{B}\phi$) and transitivity ($\mathbf{B}(\phi \vee \mathbf{B}\phi) \to \mathbf{B}\phi$) axioms, and the monotonicity rule (if $\phi \to \psi$ is provable, then so is $\mathbf{B}\phi \to \mathbf{B}\psi$). So, we see that $\mathcal{D}$ is so-called non-normal, i.e., the axiom K: $\mathbf{B}(\phi \to \psi) \to (\mathbf{B}\phi \to \mathbf{B}\psi)$ is not valid in it. Kripke-style models are introduced for $\mathcal{D}$, and their soundness and completeness proved; as a corollary, $\mathcal{D}$ is decidable. It is noted that the same arguments can be carried out when T is taken to be the classical predicate logic. Characterizing the modal logic of deductive closure of other logical systems remains an open problem.

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