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Crabbé, Marcel

On the notion of substitution. (English)

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http://www3.oup.co.uk/igpl/

Substitution has widely appeared in most areas of mathematics, from algebra to logic to computer science. In this paper, an axiomatization for the concept of substitution is given. Typical models for the substitution are taken to be two-sorted structures whose elements are either of sort **variable** or of sort **term**, with a primitive relation $\in_{\mathrm{vl}}$, where $x \in_{\mathrm{vl}} t$ is interpreted as "x is a variable of t", and a primitive operation Sub, where $Sub(x, t, s) = u$ is interpreted as "u results from t by substituting x for s". Such a model is called *logos* if it contains infinitely many variables and every term has only finitely many variables. Note that this infiniteness property is not first-order axiomatizable. Also note that a variable is a term whose only variable is the term itself.
A substitution logos is any infinite set of variables with terms over a languages augmented with those variables, where $\in_{\mathrm{vl}}$ and Sub are interpreted as usual. The completeness theorem states that "conditional equations" are valid in every logos iff they are valid in every substitution logos.
Also, simultaneous substitution and some other properties of the substitution are studied in the paper. The paper is well organized and clearly written, and could be of interest for non-experts even. Two minor typos: $M[\rightarrow := \overrightarrow{x}]n$ in Proposition 3.5 (page 118 line -2) should be $M[\overrightarrow{x} := \overrightarrow{N}]$, and L in the Proof of Corollary 4.2 (page 122 line 8) should be LC.

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Keywords : axiomatization for the concept of substitution; two-sorted structures; substitution logos
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