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Decidability, undecidability, and PSPACE-completeness of the twins property in the tropical semiring. (English summary)

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Introduction: "Weighted finite automata over the tropical semiring (WFA) are studied under various names in the literature, e.g. distance, finance, or cost automata. They have also appeared in various contexts: logical problems in formal language theory (star height, finite power property, star problem for traces), study of dynamics of some discrete event systems (DES), automatic speech recognition, and database theory.

"To achieve efficient implementations, one is interested in deterministic (sequential) WFA [M. Mohri, Comput. Linguist. **23** (1997), no. 2, 269–311; MR1713412 (2000e:68174)] which rises the determinization (sequentiality) problem: decide (constructively) whether some given min-plus automaton admits a deterministic equivalent. Its decidability was shown in 2004 by Klimann, Lombardy, Mairesse, and Prieur for finitely ambiguous WFA and recently by Lombardy and the author for polynomially ambiguous (cycle unambiguous) WFA. Despite this progress, the determinization problem is considered as wide open.

"In 1997, Mohri presented an imperfect algorithm. The algorithm tries to construct some deterministic equivalent by a generalization of the power set construction. If the algorithm terminates on some given WFA $\mathcal{A}$, then it constructs a deterministic equivalent. For many WFA, in particular for many WFA in practical applications, the algorithm successfully constructs a deterministic equivalent. However, the existence of a deterministic equivalent is necessary but not sufficient for the termination, and for many WFA, the algorithm does not terminate even if a deterministic equivalent exists.

"To study the termination of his algorithm, Mohri adapted the notion of the twins property which is a sufficient condition for the termination of his algorithm. Mohri's algorithm and the notion of the twins property became a popular object, see e.g. [C. Allauzen and M. Mohri, J. Autom. Lang. Comb. **8** (2003), no. 2, 117–144; MR2000615 (2004f:68071); D. Kirsten, Theor. Inform. Appl. **42** (2008), no. 3, 553–581; MR2434035 (2010a:68083); D. Kirsten and I. Mäurer, J. Autom. Lang. Comb. **10** (2005), no. 2-3, 287–312; MR2285332 (2007i:68064); M. Mohri, in *Handbook of weighted automata*, 213–254, Monogr. Theoret. Comput. Sci. EATCS Ser., Springer, Berlin, 2009; MR2777732] or [B. Borchardt, Acta Cybernet. **16** (2004), no. 4, 509–544; MR2104071 (2005f:68067); M. Büchse, J. May and H. Vogler, J. Autom. Lang. Comb. **15** (2010), no. 3-4, 229–254; J. May and K. Knight, in *HLT-NAACL '06 Proceedings of the Main Conference on Human Language Technology Conference of the North American Chapter of the Association of Computational Linguistics*, 351–358, Assoc. Comput. Linguist., Stroudsburg, PA, 2006, doi:10.3115/1220835.1220880] for a generalization to trees.

"For cycle unambiguous WFA, the twins property is decidable in polynomial time. In general, the decidability of the twins property remained open. It was rather conjectured that the *un*decidability of the twins property follows easily from a result by Krob from 1994 which says that the semantic equivalence of two given WFA is undecidable. Indeed, we will observe that a problem closely related to the twins property (to decide whether two given states are twins) is undecidable.

"As our main result, we will show that the twins property is decidable and PSPACE-complete."

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.