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Recognisability for algebras of infinite trees. (English summary)

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From the introduction: "This paper provides a first step in the development of an algebraic theory for recognisability of classes of infinite trees. Inspired by the work of Z. Ésik and P. Weil [Theoret. Comput. Sci. **340** (2005), no. 2, 291–321; MR2150756 (2006c:68129)] on preclones, we define suitable algebras of infinite trees called *ω-hyperclones*. We can show that every regular language is recognised by some homomorphism into such a (finitary, path-continuous) ω-hyperclone.

"The proof is performed in two steps. First, we define a special class of ω-hyperclones called *path-hyperclones* that directly correspond to tree automata. The problem with path-hyperclones is that their definition is not axiomatic, but syntactic. That is, given an arbitrary ω-hyperclone we cannot tell from the definition whether or not this ω-hyperclone is isomorphic to some path-hyperclone.

"In the second step, we therefore give an algebraic characterisation of the main properties of such path-hyperclones (they are path-continuous). Using this result we can transfer our characterisation from path-hyperclones to path-continuous ω-hyperclones.

"Finally, we prove that the class of path-continuous ω-hyperclones is closed under products and a certain power-set operation. From these results we can deduce a second (equivalent) version of our main theorem: recognisability by finitary path-continuous ω-hyperclones is the same as definability in monadic second-order logic."

From the conclusions: "[W]e do not believe that the framework we have set up is in its final form. The algebras we use (path-continuous ω-hyperclones) are far too complicated. In particular,

(1) we use infinitely many sorts,
(2) there are too many operations . . . , and
(3) the definition of path-continuity is too complex.

"Finally, a long-term goal would be the development of a theory of pseudo-varieties of path-continuous ω-hyperclones and a corresponding structure theory. But, before embarking upon such a project, it seems advisable to wait until we have found the 'right' definition for our algebras."

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.