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**Semigroups with $I$-quasi length and syntactic semigroups.** (English summary)
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For a semigroup $S$ and a subset $L \subseteq S$, the syntactic congruence of $L$ is the relation

$$\rho_L = \{(x,y) \in S \times S \mid \forall u, v \in S^1, uxv \in L \iff uyv \in L\}.$$

The quotient semigroup $S/\rho_L$ is called the syntactic semigroup of $L$. The set $L$ is called disjunctive in $S$ when $\rho_L$ is the equality (diagonal) relation on $S$. A disjunctive language over a finite alphabet $A$ is a disjunctive subset of the semigroup $A^+$ (or the monoid $A^*$). A language $L \subseteq A^+(A^*)$ is $f$-disjunctive if each $\rho_L$-class is finite, and is called $qf$-disjunctive if each $\rho_L$-class other than $\mathbf{W}(L)$ (the residue of $L$) is finite, where $\mathbf{W}(L) = \{x \in S \mid S^1 x S^1 \cap L = \varnothing\}$. A semigroup $S$ is called a semigroup with $I$-quasi-length if $I$ is an ideal of $S$, and there exists a mapping $l: I^\complement \to \mathbb{N}$ such that $l(xy) \geqslant l(x) + l(y)$ holds for all $x, y, xy \in I^\complement$. If $l$ is a partial homomorphism (i.e., satisfies $l(xy) = l(x) + l(y)$ for all $x, y, xy \in I^\complement$) then $S$ is called a semigroup with $I$-length.

The authors give some characterizations of semigroups with (0-)quasi-length, and also provide some necessary and sufficient conditions for a class of semigroups—including finitely generated semigroups with quasi-length, finite $\pi$-groups, finite nil-extensions of finite inverse semigroups—to be syntactic. A semigroup is called syntactic if it is isomorphic to the syntactic semigroup of a language. The authors claim that their results answer completely the question "What kind of semigroups can be isomorphic to the syntactic semigroup of an $f$-disjunctive (respectively, $qf$-disjunctive) language?"

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