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On nested simple recursion. (English summary)
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For primitive recursive ternary functions $s_1, s_2, \ldots, s_n$ and the binary function f, let $f_0(x,y)=0$, and

$$f_1(x,y)=f(x,s_1(x,y,f_0(x,y))),$$
$$f_2(x,y)=f(x,s_2(x,y,f_1(x,y))),$$
$$\vdots$$
$$f_n(x,y)=f(x,s_n(x,y,f_{n-1}(x,y))).$$

If g and h are primitive recursive functions, then the function f is called definable by nested simple recursion, if f satisfies the following identities:

$$f(0,y)=g(y), \quad \text{and}$$
$$f(x+1,y)=h(x,y,f_1(x,y),f_2(x,y),\ldots,f_n(x,y)).$$

In this paper, it is proved that if f is definable by nested simple recursion, as above, then f is primitive recursive. This result was first proved by R. Péter in [Math. Ann. **110** (1935), no. 1, 612–632; MR1512957] (also presented by him in [*Recursive functions*, Third revised edition. Translated from the German by István Földes, Academic Press, New York, 1967; MR0219414 (36 #2496)]). In the author's opinion, the formalization of Péter's proof "in fragments of Peano Arithmetic is far from obvious", and he hopes that "this new presentation [proof] of the old result is much simpler and also easier to follow". The author also claims in the abstract of the paper that his new proof "can be easily formalized in small fragments of Peano Arithmetic"; but no indication appears inside the paper's body, and it is not clear which small fragment of Peano Arithmetic is able to formalize the author's new proof.

The paper has no single definition, lemma, or theorem, but three proofs. So, it needs reading from the start to the end, to see what is proved and where. The reader has to figure out (and sometimes guess) what is going on, and how it is being done. No exact or rigorous definition of nested simple recursion appears, and the arguments of the whole paper deal only with the following instance (of the above-mentioned more general scheme):

$$f(0,y)=g(y),$$
$$f(x+1,y)=h(x,f(x,s_1(x,y)),f(x,s_2(x,y,f(x,s_1(x,y)))),y);$$

it is mentioned at the end of the paper that the general case can be directly reduced to the above case, in a quite straightforward way. *Saeed Salehi*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.