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Generalized contexts and n-ary syntactic semigroups of tree languages. (English summary)

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When it was announced in [M. Nivat and A. Podelski, Bull. Eur. Assoc. Theor. Comput. Sci. EATCS **38** (1989), 186–190; Zbl 0677.68066] that definite tree languages are characterizable by (their syntactic) monoids, almost nobody was surprised, as it seemed a natural generalization of definability of definite (string) languages by monoids. The fact that the proof was flawed, and indeed the family of definite tree languages is not definable by monoids or semigroups, was shown much later by the reviewer in 2005 (here is a concrete counterexample in the terminology of the above-cited paper: http://saeedsalehi.ir/images/mondef.jpg).

In the present paper, the authors define n-ary syntactic semigroups (and monoids) by introducing the notion of n-ary contexts, and develop a variety theory for these structures. Note that for $n = 1$, one gets the usual (syntactic) semigroups (monoids). For any variety of n-ary semigroups (or monoids) the family of tree languages whose syntactic n-ary semigroups (or monoids) belong to this variety is a variety of tree languages. But not any variety of tree languages is definable this way. The authors characterize the variety of definite tree languages by 2-ary semigroups. This shows that the defining power of n-ary semigroups (monoids) is between that of (universal) algebras and that of ordinary (1-ary) semigroups (monoids).

The paper is clearly written, and contributes some interesting and useful results to the subject. It opens a new door to the scientific community in the field, and will surely be highly cited in future works. *Saeed Salehi*