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Fragments of first-order logic over infinite words. (English summary)

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From the introduction: “For infinite words, the expressive power of FO^2 [first-order logic with two variables] is not equal to Δ_2 [which is $\Sigma_2 \cap \Pi_2$], since saying that letters a and b appear infinitely often, but c only finitely many times is FO^2 -definable, but there is neither a Σ_2 -formula nor a Π_2 -formula specifying this language.

“We introduce two generalizations of the usual Cantor topology for infinite words. One of our first results is a characterization of Σ_2 -definability for languages in Γ^∞ [the set of finite and infinite words over Γ]. This characterization consists of two components: The first one is an algebraic property of the syntactic monoid and the second part is requiring that L [the language] is open in some alphabetic topology. Both properties are decidable.

“Our second result is that a regular language is FO^2 -definable if and only if its syntactic monoid is in the variety DA. (The result is surprising in the sense that it contradicts a statement in [T. Wilke, *Classifying discrete temporal properties*, Habilitationsschrift, Univ. Kiel, 1998].) In addition, we show that a language is definable in FO^2 if and only if it is closed in some further refined alphabetic topology and if it is weakly recognizable by a monoid in DA. In particular, weak recognition and strong recognition do not coincide for the variety DA. This seems to be a new result as well. We also contribute language theoretic constraints on the letters which occur infinitely often.

“Other results of our paper are the characterization of $\text{FO}^2 \cap \Sigma_2$ as the class of unambiguous polynomials and of Δ_2 in terms of unambiguous polynomials in some special form and also in terms of deterministic languages. It follows already from this description that Δ_2 is a proper subset of FO^2 . Furthermore, we show that the equality of FO^2 and Δ_2 holds relativized to some fixed set of letters which occur infinitely often. If this set of letters is empty, we obtain the situation for finite words as a special case. Finally, we relate topological constructions such as *interior* and *closure* with membership in the fragment under consideration.”

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.