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A potential subtlety concerning the distinction between determinism and nondeterminism.

Infinity, 119–129, *Cambridge Univ. Press*, *Cambridge*, 2011.

First we quote from the first section of the paper: "[O]ur intention in this chapter is to illustrate a potentially subtle aspect of the distinction between determinism and nondeterminism. This subtlety is the possibility of coding arbitrary information into time in such a way that a *specific* deterministic process computes precisely that information (as additional output).

"Given any specific set of physical laws, we produce a Turing program e_0 with the following feature. First, the output of the program e_0 (by virtue of its format) must be a finite binary sequence s, but there may be no output generated. Let t be any nonempty finite binary sequence .... Then by 'extending time' and preserving all the specified physical laws, one can arrange that in the 'new' universe the program e_0 generates as additional output exactly the chosen string t (so that in the case in which no output was initially generated by the program, in the universe the total output generated is exactly t; otherwise, the total output generated is exactly s appended by the sequence t). From the perspective of the inhabitants of the universe (i.e., our perspective), passing from the initial universe to the extension is not an observable change because all laws have been preserved; more precisely, the initial universe is not an observable entity in the extension. The program e_0 is explicit; it only depends (as it must) on the specification of physical laws—if that specification is simple, then so is the program. We are implicitly assuming that time is infinite in the idealized universe and so the 'extension' is a nonstandard extension (which again is the only possibility). In fact, for the construction of e_0 we give, the program e_0 generates no output within our (idealized) universe.

"Our construction of e_0 actually gives a Turing program e_0^* that witnesses a more dramatic version of the property just discussed, and it is this program that is the basis for our claim of a potentially subtle aspect to the distinction of determinism versus nondeterminism. We describe e_0^* first more formally and then informally.

"If in a given universe no output is generated by e_0^* with input 0, then by extending time one can arrange that any specified finite binary sequence is now the output of e_0^* acting on input 1. Otherwise, in the given universe, e_0^* computes a finite (positive) integer N_0 from input 0, and for any specified finite binary sequence, by extending time one can arrange that for some input $i < N_0$, the output of e_0^* is the given sequence. In this situation, the indicated bound N_0 on potential inputs *cannot change* once it has been calculated, and for any input $n < N_0$, if e_0^* generates output from this input, then e_0^* must generate output acting on all inputs $k < n$ and *none* of these outputs can be changed by extending time.

"Informally, in any given universe the program e_0^* produces either no output or an ordered library of binary sequences together with an upper bound for the size of the library. In the former case, by extending time one can arrange that *any* binary sequence is added to the library as the first (and only) entry. In the latter case, by extending time, the upper bound cannot change, sequences cannot be deleted from the library, and *any* binary sequence can be added to the library both as the last entry and as the unique additional entry.

"How does e_0^* relate to the distinction of determinism versus nondeterminism? One can view the inputs to e_0^* as the 'initial conditions' needed to derive the outcome of a physical process based on the physical evolution of that process. For *any* physical process there is an input from which e_0^* can compute the outcome of that physical process given sufficient time. Moreover, and this is the key point, the size of the input space is *bounded* and that bound is *independent* of the physical process to be analyzed. In particular, modulo a fixed offset, the input space is *much simpler* than the physical process to be analyzed.

"If one attempts to counter any possible implication of the existence of e_0^* for the distinction of determinism versus nondeterminism by rejecting all meaning for infinite time even in an idealized sense, then one clearly has far more serious issues to contend with than the distinction between determinism and nondeterminism in the physical universe, more specifically, what is the meaning of nondeterminism in a specific finite setting?"

Fortunately, the other sections of the paper are more mathematical, presenting a technical theorem (below), though after presenting the main result of the paper, the author does not discuss its relation with the "subtle distinction of determinism vs. nondeterminism" anymore (they are only mentioned in the first section, quoted above). The main and technical result of the paper, which could be of interest even without deriving the above-mentioned conclusions, is:

For a recursive theory T extending PA there exists some natural number e_T such that for all countable models $\mathcal{M} \models T$, if $s = \text{Output}(e_T : \mathcal{M})$ and if t is a definable (with parameters) binary sequence of $\mathcal{M}$ such that s is a proper initial segment of t, then there exists a countable model $\mathcal{N} \models T$ such that $\mathcal{M}$ is a proper initial segment of $\mathcal{N}$ and $\text{Output}(e_T : \mathcal{N}) = t$.

The notion of $\text{Output}(e : \mathcal{M})$ is never defined rigorously in the paper, but one can guess that it means the output of the program with code e (the input is not specified) within the universe $\mathcal{M}$; when $\mathcal{M}$ is the standard model of arithmetic $\mathbb{N}$, then $\text{Output}(e : \mathbb{N})$ means the usual output (within our idealized universe, as the author puts it).

The paper (except for its first section) is devoted to proving the above theorem, and we have outlined the first section of the paper, so that the readers can decide for themselves if the physical-philosophical conclusions make any sense, or at least, if they can be derived from the above theorem.

{For the entire collection see MR2850464 (2012e:00007)}

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