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Ishihara, Hajime (J-AIST-SIF);
Schuster, Peter [Schuster, Peter M.] (4-LEED-PM)
On the contrapositive of countable choice. (English summary)
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The axiom of choice can be read as $\forall x \exists y R(x, y) \to \exists f \forall x R(x, f(x))$. The authors focus on the contrapositive of countable (restricted to numbers) choice: CCC: $\forall f \exists x P(x, f(x)) \to \exists x \forall y P(x, y)$. It is known [S. Berardi, Ann. Pure Appl. Logic **139** (2006), no. 1-3, 185–200; MR2206255 (2006m:03030)] that CCC follows from double negation elimination for Σ_2-formulas: Σ_2-DNE: $\neg\neg\exists x \forall y P(x, y) \to \exists x \forall y P(x, y)$, and also that CCC implies the law of excluded middle for Σ_1-formulas: Σ_1-LEM: $\exists x P(x) \vee \forall x \neg P(x)$. It was conjectured in [op. cit.] that CCC lies strictly in between those two principles.

It is shown in the present paper that by restricting P to quantifier-free predicates, CCC becomes equivalent to Σ_2-DNE in Elementary Intuitionistic Analysis **EL**. And when the function f is limited to recursive functions and the predicate P to recursive predicates, then again CCC is equivalent to Σ_2-DNE in Heyting Arithmetic **HA**.

At the end, the authors show that the decidable predicates of **HA** plus the (extended) Church thesis are recursive. It was already shown by Z. Marković (much earlier in the same journal) [Math. Logic Quart. **39** (1993), no. 4, 531–538; MR1270397 (95f:03103)] (and by D. de Jongh) that decidable predicates of **HA** (without adding Church's thesis) are recursive (Δ_1-)predicates. And the reviewer gave a proof-theoretic proof for this fact and generalized that result for some other intuitionistic arithmetical theories in [Rep. Math. Logic No. 36 (2002), 55–61; MR1983981 (2004c:03082)]. *Saeed Salehi*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.