

MR2511883 (2010c:68062) [68Q42](#) ([03F15](#))

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The Hydra battle and Cichon's principle. (English summary)

Appl. Algebra Engrg. Comm. Comput. **20** (2009), *no. 2*, 133–158.

It was L. Kirby and J. Paris [*Bull. London Math. Soc.* **14** (1982), no. 4, 285–293; [MR0663480 \(83j:03096\)](#)] who first showed that the Battle of Hercules and the Hydra terminates, and this very fact cannot be proved in Peano arithmetic (PA). The first term rewriting system for describing this battle was presented by N. Dershowitz and J.-P. Jouannaud [in *Handbook of theoretical computer science. Vol. B*, 243–320, Elsevier, Amsterdam, 1990; *Zbl* 0900.68283; see [MR1127185 \(92d:68002\)](#)] and denoted by \mathcal{H} . It was shown that \mathcal{H} terminates, with a reduction proof that used the Howard-Bachmann ordinal as its order type. Cichoń then asked: Must any termination reduction for \mathcal{H} be of Howard-Bachmann ordinal order type? Then came H. Touzet [in *Mathematical foundations of computer science, 1998 (Brno)*, 267–276, *Lecture Notes in Comput. Sci.*, 1450, Springer, Berlin, 1998; [MR1684070 \(2000a:68073\)](#)] who proved that the Battle of Hercules and the Hydra can be formulated as a term rewriting system whose order type of termination is ε_0 (the first fixed point of $\omega^x = x$, the proof theoretic ordinal of PA). In this paper the author answers the question of Cichoń negatively, by providing a termination proof for \mathcal{H} of (reduction) order type ε_0 .

Reviewed by [Saeed Salehi](#)

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