**MR2499430 (2010a:03037)** 03D05 (03D35 03F25)
**Ganea, Mihai** (1-BOST-Q)

**Arithmetic on semigroups. (English summary)**

*J. Symbolic Logic* **74** (2009), *no. 1,* 265–278.

This paper is on the theory of concatenation (or strings); thus the title is misleading, as it could have read "arithmetic on strings" or "arithmetic on free semigroups". Consider the theory $TC$ of Grzegorczyk with the following axioms, where $*$ stands for concatenation:

$$TC_1\colon x*(y*z)=(x*y)*z;$$
$$TC_2\colon x*y{=}u*v\Rightarrow$$
$$\exists\alpha[(x{=}u\,\&\,y{=}v)\vee(x*\alpha{=}u\,\&\,y{=}\alpha*v)\vee(x{=}u*\alpha\,\&\,\alpha*y{=}v)];$$
$$TC_3\colon a\neq x*y; \qquad TC_4\colon b\neq x*y; \qquad TC_5\colon a\neq b.$$

Note that $a$ and $b$ are constant (atomic) strings. Grzegorczyk asked whether Robinson's arithmetic $Q$ was interpretable in $TC$. A. Visser (2009) and V. Švejdar (2009) proved that $Q$ is indeed interpretable in $TC$; and the author gives another proof in this paper. Another theory of concatenation, introduced by A. Tarski and A. Mostowski and R. Robinson (1968), is considered in this paper. The axioms of $F$ (in the above language of $TC$) are as follows:

$$F_1\colon x*(y*z)=(x*y)*z;$$
$$F_2\colon x*y=x*z\Rightarrow y=z;$$
$$F_3\colon x*z=y*z\Rightarrow x=y;$$
$$F_4\colon x*a\neq y*b\;\&\;\forall z[z=a\vee z=b\vee\exists x(z=x*a\vee z=x*b)].$$

It was claimed (presumably by A. Tarski and W. Szmielewa) that $F$ interprets $Q$ but, as the author mentions, a printed proof has not appeared. In the paper under review, the author proves this claim by a series of interpretations, and using some modern techniques and methods. So, the puzzle (pointed out by A. Visser) remains as to what was the original proof of the interpretability of $Q$ in $F$.

Reviewed by *Saeed Salehi*

## References

1. J. Bennett, *On spectra*, Ph.D. thesis, Department of Mathematics, Princeton University, 1962.
2. V. Ča či 'c, P. Pudl 'ak, G. Restall, A. Urquhart, and A. Visser, Decorated linear order types and the theory of concatenation, LGPS preprint 258 (October 4, 2007), available online at http://www.phil.uu.nl/preprints/preprints/PREPRINTS/preprint258.pdf.
3. J. Corcoran, W. Frank, and M. Maloney, String theory, this Journal, vol. 39 (1974), no. 4, pp. 625–636. MR0398771 (53 #2622)
4. K. De Bouv'ere, Logical synonymity, *Indagationes mathematicae*, vol. 27 (1965), pp. 622–629.

MR0184845 (32 #2316)

5. M. Ganea, *Arithmetic without numbers*, Ph.D. thesis, Department of Philosophy, University of Illinois at Chicago, 2005.
6. A. Grzegorczyk, Undecidability without arithmetization, *Studia Logica*, vol. 79 (2005), no. 1, pp. 163–230. MR2135033 (2005m:03088)
7. A. Grzegorczyk and K. Zdanowski, Undecidability and concatenation, *Andrzej Mostowski and foundational studies* (A. Ehrenfeucht, V.W. Marek, and M. Srebrny, editors), IOS Press, 2008, pp. 72–91. MR2422681 (2009i:03042)
8. P. H 'ajek and P. Pudl 'ak, *Metamathematics of first-order arithmetic*, Springer, 1993. MR1219738 (94d:03001)
9. H. Hermes, Semiotik, Eine Theorie der Zeichengestalten als Grundlage f "ur Untersuchungen von formalisierten Sprachen, *Forschungen zur Logik und zur Grundlage der exakten Wissenschaften, n.s.*, vol. 5 (1938), pp. 5–22, Leipzig.
10. D. Hilbert and P. Bernays, *Grundlagen der Mathematik*, vol. I, Springer, 1934.
11. J.P. Jones and J.C. Shepderson, Variants of Robinson's essentially undecidable theory *R*, *Archive for Mathematical Logic*, vol. 23 (1983), pp. 61–64. MR0710365 (85d:03088)
12. W. Quine, Concatenation as a basis for arithmetic, this Journal, vol. 11 (1946), no. 4, pp. 105–114. MR0018618 (8,307b)
13. V. Švejdar, An interpretation of Robinson Arithmetic in its Grzegorczyk's weaker variant, *Funda- menta Informaticae*, vol. 81 (2007), pp. 347–354. MR2372699 (2008j:03093)
14. , Weak theories and essential incompleteness, *The Logica Yearbook 2007: Proceedings of the Logica '07 International Conference, Hejnice, Czech Republic* (Praha) (M. Peliš, editor), Filosofia, 2008, to appear.
15. , On interpretability in the theory of concatenation, *Notre Dame Journal of Formal Logic*, vol. 50 (2009), no. 1, pp. 87–95. MR2536702
16. A. Tarski, The concept of truth in formalized languages, *Logic, semantics, metamathematics* (J. Corcoran, editor), Hackett, 1983, English translation by J.H. Woodger, pp. 152–278. MR0736686 (85e:01065)
17. A. Tarski, A. Mostowski, and R. Robinson, *Undecidable theories*, second ed., North Holland, 1968.
18. A. Visser, Growing commas — A study of sequentiality and concatenation, *Notre Dame Journal of Formal Logic*, vol. 50 (2009), no. 1. MR2536701

*Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.*