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A Kleene theorem for weighted tree automata over distributive multioperator monoids. (English summary)

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From the introduction: "Kleene's Theorem on the equivalence of recognizability and rationality of languages has been extended to various discrete structures such as, e.g., trees, trace monoids, and pictures. This equivalence (or a slight modification of it) has also been proved for the weighted counterparts where the weights are taken from some semiring, i.e., for formal power series in non-commuting variables, formal power series of trees, formal power series in partially commuting variables, and picture series.

"Here we focus our attention on formal power series of trees. These are mappings from the set T_Σ of trees over some ranked alphabet Σ to some monoid $\underline{A}$ of which the elements are called weights. Given a semiring $\underline{K}$, the concept of a weighted tree automaton over $\underline{K}$ can be defined. A weighted tree automaton over $\underline{K}$ recognizes a formal power series of trees over the additive monoid of $\underline{K}$. The various versions of Kleene's result for formal power series of trees differ in the classes of semirings over which recognizability and rationality are defined. For example, the equivalence between recognizability and rationality is proved for semirings that are commutative, complete and continuous; the latter two properties are needed in order to solve systems of equations. Then this result is generalized in the sense that completeness and continuity can be dropped from the list of restrictions on the semiring; however, commutativity is required. It is needed in the proof of the fact that the class of recognizable formal power series of trees is closed under concatenation.

"In this paper, we prove Kleene's result for formal power series of trees which are recognized by weighted tree automata over distributive multioperator monoids. As a consequence of this, we can prove Kleene's result for weighted tree automata over *arbitrary* (i.e., not necessarily commutative) semirings."

Reviewed by *Saeed Salehi*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.