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MR2217638 (2006m:08007) 08A30
Kearnes, Keith A. (1-CO)
Congruence lattices of locally finite algebras. (English summary)
Algebra Universalis **54** (2005), *no. 2,* 237–248.

Perhaps one of the most famous open problems of universal algebra, due to E. T. Schmidt, is whether every finite lattice is isomorphic to the congruence lattice of a finite algebra. The author begins the paper under review with a question of J. Schmerl: Do we know of a finite lattice that is isomorphic to the congruence lattice of an infinite locally finite algebra, while we do not yet know if it is isomorphic to the congruence lattice of a finite algebra? Here the author answers a question of the opposite type, by showing that there exist finite algebras whose congruence lattices are not isomorphic to the congruence lattice of any infinite locally finite algebra. Moreover, he shows the existence of algebraic lattices that are not isomorphic to the congruence lattice of any locally finite algebra. G. Grätzer and Schmidt proved in 1963 [Acta Sci. Math. (Szeged) **24** (1963), 34–59; MR0151406 (27 #1391)] that any algebraic lattice *is* isomorphic to the congruence lattice of an algebra; so, by this new result of the paper under review, that algebra does not need to be locally finite. One other interesting result of the paper which is worth mentioning here is stated in the last example of the paper: every finite distributive lattice is the congruence lattice of a locally finite algebra of cardinality κ for any infinite κ, and infinitely many finite κ.

The paper is clearly written, addressed to experts (and sometimes to colleagues), and makes heavy use of its references, from notation and terminology to theorems and examples; in particular, familiarity with the book [D. Hobby and R. N. McKenzie, *The structure of finite algebras*, Contemp. Math., 76, Amer. Math. Soc., Providence, RI, 1988; MR0958685 (89m:08001)] is presumed.

Reviewed by *Saeed Salehi*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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