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Ésik, Z. (H-SZEG-C)

An algebraic characterization of temporal logics on finite trees. II. (English summary)

Proceedings of the 1*st International Conference on Algebraic Informatics*, 79–99, *Aristotle Univ. Thessaloniki, Thessaloniki*, 2005.

In this second part, the unordered version of the results of Part I [Z. Ésik, in *Proceedings of the 1st International Conference on Algebraic Informatics*, 53–77, Aristotle Univ. Thessaloniki, Thessaloniki, 2005; MR2186455 (2006j:03018)] is considered. Recall that trees on a ranked alphabet Σ are defined inductively to contain the constant symbols of Σ plus the terms $\sigma(t_1,\dots,t_n)$ where σ is an n-ary function symbol of Σ and $t_1,\dots,t_n$ are (already defined) trees. Unordered Σ-trees are defined to contain the constant symbols of Σ plus the structures $\sigma\{\{t_1,\dots,t_n\}\}$ where σ is an n-ary function symbol of Σ, $t_1,\dots,t_n$ are unordered trees, and $\{\{t_1,\dots,t_n\}\}$ is the multi-set containing $t_1,\dots,t_n$. So in ordered trees, one can speak of the i-th immediate successor of a node in a tree, while in unordered trees one can talk about the successors of nodes only, without regard to their order. Thus, the tuples $(t_1,\dots,t_n)$ in trees are replaced with the multisets $\{\{t_1,\dots,t_n\}\}$ in unordered trees.

Unordered tree languages can be recognized by commutative algebras; a Σ-algebra (A,Σ) is called commutative when $\sigma(a_1,\dots,a_n)=\sigma(a_{\pi(1)},\dots,a_{\pi(n)})$ holds for all $\sigma\in\Sigma$ and all $a_1,\dots,a_n\in A$, where π is a permutation over $\{1,\dots,n\}$.

Loosely speaking, the theory of (varieties of) tree languages and tree automata can be translated to unordered trees and commutative tree automata.

In this framework, the next modality does not make sense, since X_i identifying the i-th immediate successor of a node in a tree cannot be interpreted in unordered trees. Instead, the author introduces other closely related modalities like $X_{=i}(\varphi)$, interpreted as "having i immediate successors satisfying φ", and $X_{<i}(\varphi)$, interpreted as "having $<i$ immediate successors satisfying φ". Other modalities, like ef, do not need modification, as their translations from trees to unordered trees are straightforward.

In the last section, the author notes that when the multisets $\{\{t_1,\dots,t_n\}\}$ are replaced with sets $\{t_1,\dots,t_n\}$, we are dealing with idempotent unordered trees and one can consider idempotent commutative tree recognizers. An algebra is idempotent if it satisfies the equality

$$\sigma(a_1,\dots,a_m,a_1,\dots,a_1)=\sigma(a_1,\dots,a_m,a_2,\dots,a_2).$$

The idempotent versions of the results are briefly discussed.

Like the other parts [Part I, op. cit.; Part III, Z. Ésik, in *Proceedings of the 1st International Conference on Algebraic Informatics*, 101–110, Aristotle Univ. Thessaloniki, Thessaloniki, 2005; MR2186457 (2006j:03017)], this paper contains some typos and mistakes which can be corrected by a careful (and highly motivated) reader.

{For the entire collection see MR2184982 (2006f:68005)}

Reviewed by *Saeed Salehi*