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# Modal Logic of Herbrand Consistency in Weak Arithmetics

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By Skolem-Herbrand's Theorem a given set of first-order formulas is consistent iff every finite set of its Skolem instances is propositionally satisfiable. An Skolem instance of a formula results from substituting the free variables of its Skolemized form with some (Skolem) terms. This gives us a definition of Herbrand Consistency which is weaker than the usual (Hilbert-style) consistency; weaker in the sense that Hilbert Consistency implies Herbrand Consistency, but the cost of inferring Hilbert Consistency from Herbrand Consistency is of super-exponential (for Herbrand provability is a kind of Cut-Free provability).

So, in weak arithmetics (i.e., the proper subtheories of $\mathrm{I}\Delta_0 + \mathrm{Exp}$) Herbrand Consistency is a weak form of consistency, thus proving Gödel's Second Incompleteness Theorem for it in those arithmetics (i.e., proving the unprovability of Herbrand Consistency in weak arithmetics) is more difficult than the standard proofs of Gödel's theorems.

Provability Logic of an arithmetical theory is the set of modal formulas and rules which are deducible in the theory, when $\Box$ is interpreted as the provability predicate of that theory. In this paper we consider the provability logic of Herbrand provability of a weak arithmetic, namely of $\mathrm{I}\Delta_0 + \Omega_1$. We immediately note that the resulting modal logic is not normal, in the sense that the axiom $(\mathbf{K}): \ \Box(A \to B) \to (\Box A \to \Box B)$ is not valid in it. We do not yet have a full axiomatization of that modal logic, but so far we have enough axioms and rules to be able to prove a *formalized* form of Gödel's Second Incompleteness Theorem for Herbrand Consistency of $\mathrm{I}\Delta_0 + \Omega_1$.