A Semantic Look at Some Non-Classical Logics

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The Fundamental Theorem of …

- The Fundamental Theorem of Arithmetic
  $\forall n \geqslant 2\colon\ n = \prod_i \mathfrak{p}_i^{\alpha_i}$ uniquely, for some primes $\langle \mathfrak{p}_i \rangle_i$.

- The Fundamental Theorem of Calculus
  $\frac{d}{dx}\int_a^x f(y)dy = f(x)$ and $\int_a^x F'(y)dy = F(x) - F(a)$.

- The Fundamental Theorem of Algebra
  $\forall \langle a_i \rangle_{i<n}\, \exists z \in \mathbb{C}\colon\ z^n + \sum_{i<n} a_i z^i = \mathbf{0}$.

- The Fundamental Theorem of Linear Algebra
- The Fundamental Theorem of Galois Theory
- The Fundamental Theorem of …

The Fundamental Theorem of (Mathematical) Logic?

"The (Soundness and Strong) Completeness Theorem"

$$\Gamma \vdash \psi \iff \Gamma \models \psi$$

Syntax	Semantics
how to form a formula	how to interpret a formula
a derivation calculus $\mathscr{D}$	a class of models $\mathscr{C}$
$\Gamma \vdash_{\mathscr{D}} \psi$ for sentences Γ, ψ means	$\Gamma \models_{\mathscr{C}} \psi$ for sentences Γ, ψ means
there is a $\mathscr{D}$-proof of ψ from Γ	that for each $\mathcal{M} \in \mathscr{C}$,
	if $\mathcal{M} \models \Gamma$ then $\mathcal{M} \models \psi$

The Completeness of Propositional Logic

- SEMANTICS: Truth-Tables (CH. PEIRCE **1883 / 1893**).
- SYNTAX: *Principia Mathematica* (A.N. WHITEHEAD & B. RUSSELL, **1910, 1912, 1913**).
- E. POST (*Introduction to a General Theory of Elementary Propositions*, ***American Journal of Mathematics***, **1921**):
  - Language: $\neg, \vee$
  - Axioms:
    - $\neg(p \vee p) \vee p$ $\qquad p \vee p \to p$
    - $\neg q \vee (p \vee q)$ $\qquad q \to p \vee q$
    - $\neg(p \vee q) \vee (q \vee p)$ $\qquad p \vee q \to q \vee p$
    - $\neg(\neg q \vee r) \vee [\neg(p \vee q) \vee (p \vee r)]$ $\qquad (q \to r) \to (p \vee q \to p \vee r)$
    - $\neg[p \vee (q \vee r)] \vee [q \vee (p \vee r)]$ $\qquad p \vee (q \vee r) \to q \vee (p \vee r)$
  - Rule: $\dfrac{A, \ \neg A \vee B}{B}$ $\qquad$ MP $\dfrac{A, \ A \to B}{B}$

Note that $p \to q \equiv \neg p \vee q$, $\quad p \wedge q \equiv \neg(\neg p \vee \neg q)$, $\quad p \leftrightarrow q \equiv \cdots$.

Some History (?)

"Sentential Logic was created in 225 B.C.E. by the ancient Greek logician Chrysippus. That knowledge of logic was lost in the Dark Ages but was rediscovered by the French philosopher Abelard in the 12th century. The truth table system for Sentential Logic was invented in 1902 by the American logician Charles Peirce to display how the truth of some sentences will affect the truth of others. Truth tables were rediscovered independently by Ludwig Wittgenstein and Emil Post."

–Bradley H. Dowden (**2021**); *Logical Reasoning*, Open Education Resource (OER) LibreTexts Project.

https://human.libretexts.org/Bookshelves/Philosophy/Book%3A_Logical_Reasoning_(Dowden)

"syntactic ("Post-") and semantic completeness ... were first obtained by Hilbert and Bernays in **1918**." (R. Zach, *BSL* **1999**.)

The Completeness of A Sub–Propositional Logic

Implicational Logic ($\rightarrow$)

- $p \rightarrow (q \rightarrow p)$
- $[p \rightarrow (q \rightarrow r)] \longrightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$
- $[(p \rightarrow q) \rightarrow p] \longrightarrow p$
- Rule:

$$\text{MP}\ \frac{A,\ A \rightarrow B}{B}$$

Implication and Falsum ($\rightarrow, \mathbb{F}$)

- $\mathbb{F} \rightarrow p$

$$\neg q \equiv (q \rightarrow \mathbb{F}), \quad p \vee q \equiv \neg p \rightarrow q, \quad p \wedge q \equiv \neg(p \rightarrow \neg q).$$

The Completeness of Classical Logic(s)

Implication and Negation ($\rightarrow, \neg$)

- $p \rightarrow (q \rightarrow p)$
- $[p \rightarrow (q \rightarrow r)] \longrightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$
- $[\neg q \rightarrow \neg p] \longrightarrow [p \rightarrow q]$
- Rule:

$$\text{MP}\ \frac{A,\ A \rightarrow B}{B}$$

Ignoring Aristotle's Syllogistic Logic and Equality Logic ...

Predicate Logic

- D. Hilbert & W. Ackermann (book **1928**)
- D. Hilbert (M.A. **1929**)

K. Gödel (1930)

- Language: $\neg, \vee, \forall$ Write $(a \longrightarrow b) \equiv (\neg a \vee b)$
- Axioms:
  - $p \vee p \to p$
  - $q \to p \vee q$
  - $p \vee q \to q \vee p$
  - $(q \to r) \longrightarrow (p \vee q \to p \vee r)$
  - $\forall x F(x) \to F(t)$
  - $\forall x [A \vee F(x)] \to A \vee \forall x F(x)$ $x \notin \mathrm{FV}(A)$
- Rules:

$$\text{MP}\ \frac{A,\ A \to B}{B} \qquad \text{G}\ \frac{F(x)}{\forall x F(x)}$$

Note that $\exists x F(x) \equiv \neg \forall x \neg F(x)$.

The Completeness of Classical Predicate Logic

Implication, Negation, and Universal Quantifier $(\rightarrow, \neg, \forall)$

- $p \rightarrow (q \rightarrow p)$
- $[p \rightarrow (q \rightarrow r)] \longrightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$
- $[\neg q \rightarrow \neg p] \longrightarrow [p \rightarrow q]$
- $\forall x\varphi \rightarrow \varphi(x/t)$
- $\forall x(\varphi \rightarrow \psi) \rightarrow (\forall x\varphi \rightarrow \forall x\psi)$
- $\theta \rightarrow \forall x\theta \qquad x \notin \mathrm{FV}(\theta)$
- Rule:

$$\text{MP}\ \frac{A,\ A \rightarrow B}{B}$$

Second Order Classical Logic (?)

Syntax	Semantics
how to form a formula	how to interpret a formula
a derivation calculus $\mathscr{D}$	a class of models $\mathscr{C}$
$\Gamma \vdash_{\mathscr{D}} \psi$ for sentences Γ, ψ means	$\Gamma \vDash_{\mathscr{C}} \psi$ for sentences Γ, ψ means
there is a $\mathscr{D}$-proof of ψ from Γ	that for each $\mathcal{M} \in \mathscr{C}$,
	if $\mathcal{M} \vDash \Gamma$ then $\mathcal{M} \vDash \psi$

As a consequence of Gödel's *In*completeness Theorem (**1931**), there is *no* re (algorithmically generable) set of sentences that can completely axiomatize the standard second-order semantics.

L. Henkin ...

A Syntax/Semantics Game

Drinker's Paradox: $\exists x[D(x)\to\forall yD(y)]$

— R. SMULLYAN (**1978**) "drinking principle"

SYNTACTIC	SEMANTIC
$\exists x[\neg D(x)\vee\forall yD(y)]$	$\exists ?d$: $D(d)\to\forall yD(y)$
$\exists x\neg D(x)\vee\forall yD(y)$	if $\forall yD(y)$ then let $d=$ arbitrary
$\neg\forall xD(x)\vee\forall yD(y)$	if $\exists y\neg D(y)$ then let $d=$ that y
$\neg p\vee p$	in any case, $D(d)\to\forall yD(y)$

Barber's Paradox: $\neg\exists x\forall y[S(x,y)\leftrightarrow\neg S(y,y)]$

— B. RUSSELL (**1918**).

SYNTACTIC	SEMANTIC
$\forall x\neg\forall y[S(x,y)\leftrightarrow\neg S(y,y)]$	if $\forall y[S(b,y)\leftrightarrow\neg S(y,y)]$ then
$\vdots$	$\vdots$

Non-Classical Logics ...

- Many-Valued Logics
  Semantics
- Intuitionistic Logic
  Syntax
- Fuzzy Logics
  Semantics
- Sub-Classical Logics
  Syntax & Semantics

Intuitionistic Propositional Logic (IPL)

As formalized by A. HEYTING (**1930**):

- Language: $\neg, \wedge, \vee, \rightarrow$
- Axioms:
  - $p \rightarrow p \wedge p$
  - $p \wedge q \rightarrow q \wedge p$
  - $(p \rightarrow q) \rightarrow (p \wedge r \rightarrow q \wedge r)$
  - $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$
  - $q \rightarrow (p \rightarrow q)$
  - $p \wedge (p \rightarrow q) \rightarrow q$
  - $p \rightarrow p \vee q$
  - $p \vee q \rightarrow q \vee p$
  - $(p \rightarrow r) \wedge (q \rightarrow r) \rightarrow (p \vee q \rightarrow r)$
  - $\neg p \rightarrow (p \rightarrow q)$
  - $(p \rightarrow q) \wedge (p \rightarrow \neg q) \rightarrow \neg p$
- Rule:

$$\text{MP}\ \frac{A,\ \ A \rightarrow B}{B}$$

$$\boxed{\text{CPL} = \text{IPL} + (p \vee \neg p)}$$

Chapter 11
From Intuitionism to Many-Valued Logics Through Kripke Models

Saeed Salehi

Abstract Intuitionistic Propositional Logic is proved to be an infinitely many valued logic by Gödel (Kurt Gödel collected works (Volume I) Publications 1929–1936, Oxford University Press, pp 222–225, 1932), and it is proved by Jaśkowski (Actes du Congrés International de Philosophie Scientifique, VI. Philosophie des Mathématiques, Actualités Scientifiques et Industrielles 393:58–61, 1936) to be a countably many valued logic. In this paper, we provide alternative proofs for these theorems by using models of Kripke (J Symbol Logic 24(1):1–14, 1959). Gödel's proof gave rise to an intermediate propositional logic (between intuitionistic and classical), that is known nowadays as Gödel or the Gödel-Dummett Logic, and is studied by fuzzy logicians as well. We also provide some results on the inter-definability of propositional connectives in this logic.

Dedicated to Professor MOHAMMAD ARDESHIR with high appreciation and admiration.

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M. Mojtahedi et al. (eds.), *Mathematics, Logic, and their Philosophies*, Logic, Epistemology, and the Unity of Science 49, https://doi.org/10.1007/978-3-030-53654-1_11

Definition 1 (*Kripke Frames*)
A *Kripke frame* is a partially ordered set; i.e., an ordered pair $\langle K, \succcurlyeq \rangle$ where $\succcurlyeq \subseteq K^2$ is a reflexive, transitive and anti-symmetric binary relation on K. ✧

Definition 3 (*Kripke Models*)
A *Kripke model* is a triple $\mathcal{K} = \langle K, \succcurlyeq, \Vdash \rangle$, where $\langle K, \succcurlyeq \rangle$ is a Kripke frame equipped with a persistent binary (satisfaction) relation $\Vdash \subseteq K \times \mathtt{At}$; persistency (of the relation $\Vdash$ with respect to $\succcurlyeq$) means that for all $k, k' \in K$ and $\mathfrak{p} \in \mathtt{At}$, if $k' \succcurlyeq k \Vdash \mathfrak{p}$ then $k' \Vdash \mathfrak{p}$.

The satisfaction relation can be extended to all the (propositional) formulas, i.e., to $\Vdash \subseteq K \times \mathtt{Fm}$, as follows:

- $k \Vdash \top$.
- $k \Vdash (\varphi \wedge \psi) \iff k \Vdash \varphi$ and $k \Vdash \psi$.
- $k \Vdash (\varphi \vee \psi) \iff k \Vdash \varphi$ or $k \Vdash \psi$.
- $k \Vdash (\neg \varphi) \iff \forall k' \succcurlyeq k (k' \nVdash \varphi)$.
- $k \Vdash (\varphi \rightarrow \psi) \iff \forall k' \succcurlyeq k (k' \Vdash \varphi \Rightarrow k' \Vdash \psi)$. ✧

Lemma 1 (A Tautology in n-Valued Logics)
For any $n > 1$, the formula $\bigvee\!\!\!\bigvee_{i<j\leqslant n}(\mathfrak{p}_i \rightarrow \mathfrak{p}_j)$ is a tautology in any n-valued logic in which the formula $(\mathfrak{p} \rightarrow \mathfrak{p}) \vee \mathfrak{q}$ is a tautology.

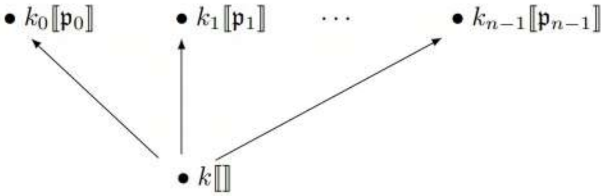
Proof In an n-valued logic, the $n+1$ atoms $\{\mathfrak{p}_0, \mathfrak{p}_1, \ldots, \mathfrak{p}_n\}$ can take n values. So, under a valuation function, there should exist some $i < j \leqslant n$ such that $\mathfrak{p}_i$ and $\mathfrak{p}_j$ take the same value, by the Pigeonhole Principle. Since $(\mathfrak{p} \rightarrow \mathfrak{p}) \vee \mathfrak{q}$ is a tautology, then the formula $\bigvee\!\!\!\bigvee_{i<j\leqslant n}(\mathfrak{p}_i \rightarrow \mathfrak{p}_j)$ should be mapped to the designated value by all the valuation functions. ❑

The lemma implies that the formula $(A \rightarrow B) \vee (A \rightarrow C) \vee (B \rightarrow C)$ is a tautology in the classical propositional logic; this formula is not a tautology in the intuitionistic (or even Gödel-Dummett) propositional logic.

Theorem 1 (Gödel 1932: IPL Is Not Finitely Many Valued)
Intuitionistic propositional logic is not finitely many valued.

Proof By Lemma 1 it suffices to show that for any $n > 1$, $\bigvee\!\!\!\!\bigvee_{i<j\leqslant n}(\mathfrak{p}_i \to \mathfrak{p}_j)$ is not a tautology in IPL. Consider the Kripke model $\mathcal{K} = \langle K, \succcurlyeq, \Vdash\rangle$ with

$K = \{k, k_0, k_1, \ldots, k_{n-1}\}$,
$\succcurlyeq = \{(k_i, k) \mid i < n\} \cup \{(k_i, k_i) \mid i < n\} \cup \{(k, k)\}$, and
$\Vdash = \{(k_0, \mathfrak{p}_0), (k_1, \mathfrak{p}_1), \ldots, (k_{n-1}, \mathfrak{p}_{n-1})\}$.



For any $i < n$ we have $k_i \Vdash \mathfrak{p}_i$, and also $k_i \nVdash \mathfrak{p}_j$ for any $j > i$. So, $k_i \nVdash \mathfrak{p}_i \to \mathfrak{p}_j$ for any $i < j \leqslant n$; which implies that $k \nVdash \bigvee\!\!\!\!\bigvee_{i<j\leqslant n}(\mathfrak{p}_i \to \mathfrak{p}_j)$. ❑

K. GÖDEL (1932)

- A.S. Troelstra's Introductory Note in GÖDEL's Collected Works is misleading and mistaken.

- GÖDEL's sentences are

$$F_n = \bigvee_{1 \leqslant i < j \leqslant n} (p_i \leftrightarrow p_j)$$

and the proof works for logics which prove $(p \leftrightarrow p) \vee q$, such as IPL.

- GÖDEL's (Fuzzy Propositional) Logics: $\mathrm{GL}_n = \mathrm{IPL} + F_n$ (for $n > 2$!).

- We have $\mathrm{IPL} \subsetneq \cdots \subsetneq \mathrm{GL}_{n+1} \subsetneq \mathrm{GL}_n \subsetneq \cdots \subsetneq \mathrm{GL}_3 = \mathrm{CPC}$.

K. Gödel (**1932**)'s Proof

- Obviously, $\mathrm{GL}_{n+1} \subseteq \mathrm{GL}_n$ since $F_n \vdash F_{n+1}$ (by $a \to a \vee b$).
- Also, $F_3 \vdash (\top \leftrightarrow a) \vee (a \leftrightarrow \bot) \vee (\top \leftrightarrow \bot)$ $[\equiv a \vee \neg a]$.
- Gödel uses a finitely many-valued semantics to show $\mathrm{IPL} \nvdash F_n$.

  ▶ Thus, IPL is not n-many-valued (for any $n > 2$), since F_n is a tautology in any n-many-valued logic.

- Fuzzy version of Gödel's m-valued semantics ($\mathcal{S}_m$) is:
  - ▶ Values: $\{0, \frac{1}{m-1}, \frac{2}{m-1}, \cdots, \frac{m-2}{m-1}, 1\}$
  - ▶ Truth: 1, Falsity: 0
  - ▶ $v(a \wedge b) = \min\{v(a), v(b)\}$
  - ▶ $v(a \vee b) = \max\{v(a), v(b)\}$
  - ▶ $v(\neg a) = v(a \to \mathbb{F})$
  - ▶ $v(a \to b) = \begin{cases} 1 & \text{if } v(a) \leqslant v(b), \\ v(b) & \text{if } v(a) > v(b). \end{cases}$

$$\begin{array}{l} \vDash_{\mathcal{S}_m} \mathrm{IPL} + \{F_n\}_{n>m} \\ n \leqslant m \Longrightarrow \nvDash_{\mathcal{S}_m} F_n \\ \hline \therefore \quad \mathrm{IPL} \nvDash_{\mathcal{S}_n} F_n \end{array}$$

$$a \leftrightarrow b = \begin{cases} 1 & \text{if } a = b, \\ a \wedge b & \text{if } a \neq b. \end{cases}$$

Gödel–Dummet Logic

- Gödel (Fuzzy Propositional) Logic:

$$\mathrm{GDL} = \bigcap_{n>2} \mathrm{GL}_n = \mathrm{IPL} + (p \to q) \vee (q \to p)$$

is (sound and strongly) complete with respect to

Linear (rooted) Kripke Models, and also Connected Kripke Models.

Two Semantics for One Logic!

- Fuzzy version of Gödel's infinitely many-valued semantics is:
  - Values: $[0, 1]$
  - Truth: 1, Falsity: 0.
  - $v(a \wedge b) = \min\{v(a), v(b)\}$
  - $v(a \vee b) = \max\{v(a), v(b)\}$
  - $v(a \to b) = \begin{cases} 1 & \text{if } a \leqslant b, \\ b & \text{if } a > b. \end{cases}$
  - $v(\neg a) = v(a \to 0)$

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METHODOLOGIES AND APPLICATION

Kripke semantics for fuzzy logics

Parvin Safari[1] · Saeed Salehi[1]

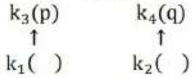
3 Conclusions

Gödel fuzzy logic is axiomatized as BL plus the axiom $\varphi \to (\varphi \& \varphi)$ of idempotence of conjunction (cf. Bendová 1999). Dummett (1959) showed that this logic can be completely axiomatized by the axioms of intuitionistic logic plus the axiom $(\varphi \to \psi) \vee (\psi \to \varphi)$. Indeed, the Gödel–Dummett logic is sound and strongly complete with respect to reflexive, transitive, connected, and persistent Kripke models. In Corollary 1, we showed that the only class of Kripke models which could be sound and (strongly) complete for a logic containing BL must contain the class of reflexive, transitive, connected and persistent Kripke models. In the other words, any logic that contains BL and is axiomatizing a class of Kripke frames/models must also contain the Gödel–Dummett logic (cf. Proposition 2). So, a Kripke-model-theoretic characterization of Gödel fuzzy logic is that *it is the smallest fuzzy logic containing the basic fuzzy logic which is sound and complete with respect to a class of Kripke frames/models*. Also, the class of reflexive, transitive, connected, and persistent Kripke models is the smallest class that can be axiomatized by a propositional fuzzy logic.

قضیه ۳.۳. در منطق گودل G، رابط فصل با استفاده از استلزام و نقیض تعریف شدنی نیست.

اثبات. مدل کریپکی $\boldsymbol{K} = \langle K, R, \vDash \rangle$ را به صورت زیر در نظر می‌گیریم:

- $K = \{k_1, k_2, k_3, k_4\}$.
- $R = \{\langle k_1, k_3\rangle, \langle k_2, k_4\rangle\} \cup \{\langle k_i, k_i\rangle | i = 1,2,3,4\}$
- $D(q) = \{k_4\}, D(p) = \{k_3\}$.



لذا با تعریف فوق $D(p\vee q) = \{k_3, k_4\}$. با استقرا روی فرمول‌های $\theta \in \mathcal{L}(\rightarrow, 0, p, q)$ نشان می‌دهیم که رابطه‌ی زیر برقرار است:

$$k_3, k_4 \in D(\theta) \Rightarrow k_1 \in D(\theta) \text{ or } k_2 \in D(\theta) \quad (\times)$$

پس برای هیچ فرمول $\theta \in \mathcal{L}(\rightarrow, 0, p, q)$ نمی‌توانیم داشته باشیم $D(\theta) = \{k_3, k_4\}$ و بنابراین

توجه داریم که مدل کریپکی ارایه شده در برهان قضیه ۳.۳ همبند است، پس مدلی مناسب برای منطق گودل G می‌باشد. ولی مدل کریپکی ارایه شده در برهان قضیه ۳.۲ همبند نیست؛ با اینحال مناسب برای منطق شهودی است.

پیوست: برهانی فازی برای قضیه ۳.۳

اثبات. به استقراء روی فرمول‌های θ در زبان منطقی $\mathcal{L}(\rightarrow, 0, p, q)$ نشان می‌دهیم که هیچ کدام از آنها معادل با فرمول $p\vee q$ (برای $p, q < 1$) نیستند؛ این مطلب برای $p, q, 0$ واضح است. پس فرض می‌کنیم که برای فرمول‌های φ, ψ داریم $\varphi \not\equiv (p\vee q)$ و $\psi \not\equiv (p\vee q)$؛ و نشان می‌دهیم که حکم زیر نیز برقرار است: $(\varphi \rightarrow \psi) \not\equiv (p\vee q)$. فرض (خلف) می‌کنیم که $(\varphi \rightarrow \psi) \equiv (p\vee q)$ و دو حالت زیر را تشخیص می‌دهیم:

۱. $p \leq q < 1$: در اینصورت $(p\vee q) = q$.

پس باید داشته باشیم $\psi \equiv (\varphi \rightarrow \psi)$. در نتیجه در این حالت باید $\psi \equiv (p\vee q)$ برقرار باشد.

Inter-Definability of Propositional Connectives

- Classical Logic: Everything is definable from
  $\{\neg, \rightarrow\}$, $\{\neg, \vee\}$, $\{\neg, \wedge\}$;
  but not from $\{\vee, \wedge, \rightarrow, \leftrightarrow\}$, $\{\neg, \leftrightarrow\}$.

- Intuitionistic Logic: Nothing is definable from the rest!

- Gödel–Dummett Logic: Only $\vee$ is definable from $\{\rightarrow, \wedge\}$,
  Two Axiomatizations for One Logic!
  $\text{GDL} = \text{IPL} + (p \vee q) \Leftrightarrow [(p \rightarrow q) \rightarrow q] \wedge [(q \rightarrow p) \rightarrow p]$;
  no other connective is definable from the rest.

Different Semantics ...

We have another semantics for Intuitionistic (Predicate) Logic (and Arithmetic, HA): KLEENE's Recursive Realizability (*JSL* **1945**).

HA is sound (but not complete; $\mathrm{HA}+\mathrm{ECT}_0$ is).

S.C. KLEENE showed that

Double Negation Shift (DNS) $\forall x \neg\neg\alpha(x) \to \neg\neg\forall x \alpha(x)$

is not realizable, and so DNS is not provable from HA

(even $\mathrm{HA}+\mathrm{ECT}_0 \nvdash \mathrm{DNS}$).

But we do not have an explicit KRIPKE Model $\mathfrak{M}$ of HA in which DNS does not hold (i.e., $\mathfrak{M} \vDash \mathrm{HA}$ and $\mathfrak{M} \nvDash \mathrm{DNS}$).

Problem (Open)

Find / Construct one such KRIPKE model. ♦

Thank You!

Thanks to

The Participants . . . . For Listening · · ·

and

The Organizers — For Taking Care of Everything · · ·